

1. Econ., with theory about decision-makers, prefer to ignore uncertainty - Statisticians aren't tempted to do so.

2) Statisticians began to consider problems: how should they "act"? What info does decision-maker need, to act reasonably? Got some results.

b) Systems analysis: how to design analysis, what info and advice to give: focus on uncertainty (not just complexity). Uses rules. (see Hitch)

c) Savage purports to solve problem!

At least, he provides way to measure subjective degrees of belief, relevant to problem. And defines clear tests of normative solution.

No ~~such~~ concept emerges corresponding to vagueness of belief; and those like Keynes, Koopman, Good, Knight who mention concept, don't predict actions.

Vagueness is incompatible with giving high odds; but to Savage, is indistinguishable from attaching uniform dist. [But this will lead to giving high odds on some propositions. Realistic.]

Need to apportion occasional unwillingness to bet giving odds on any aspect of an event; prefer not to bet.

Savage: if you will pay pS for a stake S on E , and pS for stake S on F , then if you will pay ^{at most} qS for stake S on $\bar{E}$, you must pay no more than qS for stake S on $\bar{F}$.

If $qS + pS = S$, then utility is linear on $0-S$, and $\text{prob} = p$

Or: if you pay pS for stake S on E , and $p'S < pS$ for stake S on F , then

if you pay rT for stake T on $\bar{E}$, you must be willing to pay $r'T > rT$ for stake T on $\bar{F}$.

What we can say when we see

$I > II$ is that

	E	$\bar{E}$	
I	a	b	B
II	b	a	$p^* < \bar{p}_*$

$$p^*(E) \geq \bar{p}_* = (1 - p^*)$$

$$\text{so } p^*(E) \geq \frac{1}{2}$$

$E \quad F \quad \bar{E} \quad \bar{F}$

Or from $III \quad a \quad b \quad b$

$III > IV \quad IV \quad b \quad a \quad b$

we can infer $p^*(E) \geq p_*(F)$ For a conservative, lets against

If also: $V \quad b \quad a \quad a$ an event will reveal its upper prob.

$V < VI \quad VI \quad a \quad b \quad a$ since its rank w.r.t. other probs.

we can infer: $p^*(\bar{E}) \leq p_*(F)$ [BUT IF $V > VI$: $p_*(E) \leq p_*(F) \leq p^*(E)$]

$$p^*(\bar{E}) = (1 - p^*(E)) \leq (1 - p^*(F)) = p_*(F), \text{ so } p^*(\bar{E}) \leq p_*(E)$$

It may have been US use of the A-bomb that has inhibited it since. Wouldn't we have used them in Korea? (How does Truman explain that he didn't?) At the same time, the fact that we had, convinced Americans that we would be willing to use them again, "when the time came."

If we had once used an H-bomb, we might be even more inhibited from threatening it or planning to use it than we are.

Burke & Hedges, 4th Berkeley Imp.

We have picked an action, but we want to do better.

What does it mean to "do better"?

We invent reasons for preferring one solution to another.

e.g. We can ask: Which has the greatest ²⁾ maximum risk?

b) max regret?

c) least favorable dist?

Why would we care about (c)? If A has greater LFD than B's best guess distribution, or B's most favorable dist — choice is clear.

If this is not true, but A's LFD is better than B's LFD — this is not sufficient for choice, unless BG dists and MFD's are equivalent.

In that case, A is ~~a little~~ "safer" in case of "bad luck." In particular, suppose LFD's ^{involve} ~~are~~ roughly the same distribution over events; then A is better if those "unfortunate" events occur, and about the same otherwise.

We can ask: ^{minimization of the} 2) What is the maximum loss? (instead of: LFD)

b) What is the max regret ^{for given strat} (instead of MFR)

This puts limit on: the amount of insurance that could be achieved within given set of strats

= maximum value of perfect info for given strategy.

Problem of expressing uncertainty & Savage claims all kinds can be expressed in terms of subjective probs; i.e. all kinds relevant to anyone's decisions.

Occasions when statisticians "adjusted," or "rejected" results of computations which later turned out to be right (1948, 1952 elections; see.....);

but to regard them as "fools" or "disloyal" is to ignore how much judgment goes into most of their announcements; ~~the~~ and how much worse their general record would be without it.

Savage implies: Can ignore relative probs of constant columns:

					50	50		100	
					1	2	3	4	
a	b	a	b		2	2	6	6	
b	a	b	a		2	6	2	6	
					2	3	1.4		
a					1	0	0		$\frac{1}{4}$
					0	1	0		$\frac{1}{8}$

But

					$\frac{1}{4}$	$\frac{1}{8}$		
b	a	b	a					
b	b	a	a	←				
					1	0	1	$\frac{5}{8}$
					0	1	1	$\frac{3}{4}$

r 4

~~1 1 1 1~~

1 1 0 0
1 0 1 0

1 0 0 6 6
2 0 6 2 6

1 1 0 1
1 0 1 1

25

~~50~~ 75

supple:
1, 2, 2

A B C D
1 0 0 0
0 1 0 0

A B $\bar{A}\bar{B}$

I 1 0 0
II 0 1 0

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

1 0 1 1
0 1 1 1

III 0 1 1
IV 1 0 1

$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

$A > B$

Suppose $I > II$

←

and $III < IV$

1 0 0 1
0 1 0 1

$A \cup D > B \cup D$

1 0 1 0
0 1 1 0

$A \cup C > B \cup C$

1 0 1 1
0 1 1 1

see section 141 (d) of Title 10, US Code
on VGS

	30	60		
	R	Y	B	
V	10	-10	0	$-\frac{10}{6}$
VI	0	0	0	0

Then $V > VI$

$$(p_1 \cdot 1 + p_2 \cdot (-1) + (1 - p_1 - p_2) \cdot 0)$$

$$= p_1 - p_2$$

$$\triangleright 0 \Leftrightarrow p_1 > p_2$$

Assume			
	Heads	Tails	
I	10	-10	$I = II$
II	0	0	
III	10	0	0
IV	0	10	0

$III > IV$

Need two break-even points:

a) $\begin{matrix} 1 & 0 & 0 \\ 0 & X & 0 \end{matrix} \quad p_1, p_2$

$$p_2 X = p_1 \quad p_2 = \frac{1}{X} p_1$$

b) $\begin{matrix} 0 & 1 & 1 \\ Y & 0 & Y \end{matrix}$

S: $(1 - p_1) = (1 - p_2) Y$

$(1 - p_2) = \frac{1}{Y} (1 - p_1)$

$1 - p_2 = \frac{1}{Y} - \frac{p_1}{Y}$

$p_2 = 1 - \frac{1}{Y} + \frac{p_1}{Y} = \frac{Y-1}{Y} + \frac{1}{Y} p_1$

$1 \quad 0 \quad p_1$

$0 \quad 1 \quad q = 1 - p^*$

Savage theory shows irrelevance of betting behavior.
But it is too restrictive. (see preceding page).

A normative theory, in Savage sense, is a "theory of our 'true' preferences." It is useful to the extent that such a theory is useful:

- 2) to "check" and "correct" our hasty choices.
- b) to derive complex choices from opinions of which we are confident (which we do not expect reflection to reverse).

Like establishing the truth of a proposition conditional on the truth of certain simple propositions, by showing that it is logically implied by them;

Savage approach hopes to establish a "reasonable degree of belief" in a prop. by relating it to our degrees of belief in other, more familiar, propositions.

Axioms are a more basic theory of our preferences.

Empirical: test is reflection (see Samuelson, Good, Savage. Vs. Marschak).

I say: Savage theory is inadequate theory of prefs;
it leaves out an essential dimension, treats cases differing in this dimension as alike; uncontrolled variation will lead to contradiction of theory.

Minimax "theory" rarely made claims to being normatively superior: merely "applicable."
 Neither did vN-M, for utility.

Savage is more ambitious: shows the possibility of an adequate normative theory, by the nearness of his miss.

	Heads				Tails				
	R_H	B_H	R_T	B_T	R_H	B_H	R_T	B_T	
Option A:	1	0	0	0	0	1	0	0	$\frac{1}{4} \cdot 1$
Option B:	0	0	1	0	0	0	0	1	$\frac{1}{4} \cdot 1$

(If I were told coin was Heads, I would not prefer A, though $C > D$).

A: Heads 1, Tails, 2

To call this "domination"

B: Heads 3, Tails 4

is to abolish any distinction

B does not dominate A between the Sure-thing Principle and Domination (or between

What is true is that:

consequences and prospects, or actions and states of the world). $D > C$

C	1	0	0	0	1	0	0	0	
D	0	0	1	0	0	0	1	0	(not <u>dominate</u>)

E	0	1	0	0	0	1	0	0	$F > E$
---	---	---	---	---	---	---	---	---	---------

F	0	0	0	1	0	0	0	1	
---	---	---	---	---	---	---	---	---	--

G	1	1	1	1	0	0	0	0	$G = H$
---	---	---	---	---	---	---	---	---	---------

H	0	0	0	0	1	1	1	1	
---	---	---	---	---	---	---	---	---	--

But these imply $A > B$ only by P2, not by admissibility (so long as you can distinguish between outcomes and prospects). OVER

What I want are axioms from which I can deduce that $p_* \leq p^*$ for all events.

		E	F	$\bar{E} \cap \bar{F}$
From:	I	1	0	0
	II	0	1	0

$I \geq II$; I want to deduce that $p_*(F) \leq p^*(E)$

$$\text{hence that } (1 - p^*(\bar{F})) \leq (1 - p_*(\bar{E}))$$

$$\text{or: } p^*(\bar{F}) \geq p_*(\bar{E})$$

I do not want to deduce, necessarily, from $I > II$ that $III > IV$ below: for that $p_*(F) > p^*(\bar{E})$

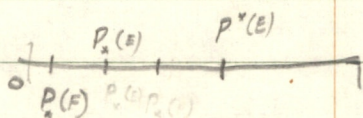
III	1	0	1	$p_*(F)$
IV	0	1	1	$p_*(\bar{E})$

I allow $III \geq IV$.

HYP: 2) Given $I > II$ and $III > IV$, then person should always act "as if" $pr(E) > pr(F)$,

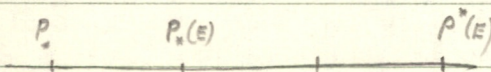
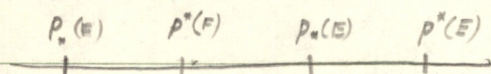
$$I > II: \quad p_*(F) < p^*(E)$$

$$III > IV \quad p_*(\bar{E}) < p^*(\bar{F})$$



If I assume conservation $\Delta = 0$; Hence
 then $I > II \Rightarrow P_*(F) < P_*(E)$

I want to allow possibility that $P_*(E) < P^*(F) < P^*(E)$



like vN-M: "We have defined 'utility' as "that variable for which the calculus of expectation is appropriate" (with probs "given")

Savage: defines prob in the same way (first, with (0,1) payoffs. ANOTHER PROPERTY OF 0,1 PAYOFFS

S-scale (permit direct estimates of probs).

I say, not that person doesn't behave "as if" he assigned probs: but "as if" he did not max expected utility on the basis of any definite probs (even in 0,1 case).

↓

I	1	0	0	0	1	0	0	0
II	0	1	0	0	0	1	0	0
III	0	0	1	0	0	0	1	0
IV	0	0	0	1	0	0	0	1
V	1	1	1	1	0	0	0	0
VI	0	0	0	0	1	1	1	1
VII	1	1	1	0	0	0	0	0
VIII	0	0	0	0	1	1	1	0

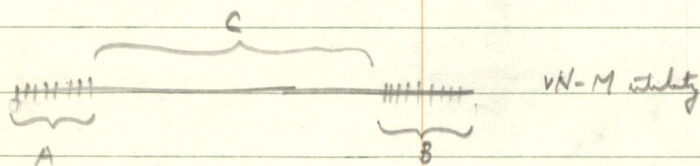
$$I = II = III = IV, \text{ and } V \neq VI \Rightarrow VII = VIII$$

De

0-1

Or, lexicographic

approx. to 0,1 payoffs:

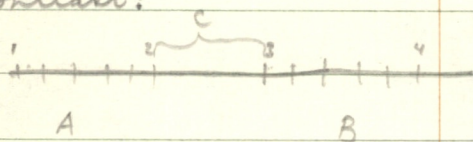


Both A and B small relative to C

So any bet with any appreciable prob. of an element of A ("war" or "defeat") $<$ one which has no A.

Any bet with appreciable prob. of an element of B $>$ any one which doesn't. ("peace; victory")
Don't "worry" much about payoffs within A or B; focus unhesitatingly on staying out of A.

Contrast:



Might prefer $(4, 2; p)$ to 3, with p sufficiently high but < 1
Might prefer 2 to $(1, 3; p)$ or even $(1, 4)$ with p sufficiently high.
but < 1 .

Do devote time + energy to problems of assessing 2 rather than 1, 4 rather than 3, not just 3 rather than 2 or 4 rather than 1.

PROBLEM: Threaten/plan to hit cities, or not? For each side, as a function of....

OR: C+G

O-1

Utility theory = Criterion theory = Objectives Theory

- O, I payoffs :
- 1) May be used by some actors in some situations
 - 2) When used, have special properties of interest
 - 3) Are often urged, because of some properties (simplicity, or implied choices) but in ignorance of others)
 - 4) May be considered, or adopted casually, without awareness of consequences.

Satisficing. Search behavior.

Value of info

Even if info has no value in a particular problem, you can choose action to get info (by observing payoffs?); anyway, you will get info, which you can keep and use "next time."

Define Consequences as "things for which the postulate of admissibility is appropriate."

Given a set of prizes & events, we can distinguish between Sure-thing Principle (or Independence Axiom) and principle of admissibility, when latter applies to "elements", events, outcomes.

P2 may not apply even when admissibility does.

Admiss. will not apply when objects are lottery tickets; or even some other times (when subject is analysing prize into prospects, ambiguous, but observer is not). But probably often does; e.g. for money prizes.

I reject extending the principle of admissibility to case of prospects or ambiguous strata as prizes. (Minor consequence: can't assign VN-M utilities to ^{ambiguous} prospects). (Major: can't always assign probs 1/2).

But may still be able to predict choices, given more data.

	1	0	0	1
	P	P	q	q

Consequences: elements 2, 6, such that

$I > II \iff III > IV$ for all E.

E		$\tilde{E}$	E		$\tilde{E}$
I	a	a	III	b	b
II	b	b	IV	b	b

(Not true - I say - for prospects, strategies, like I-IV; assumption that these are "like" consequences is P2 or Independence Post.).

What should you do when you don't know the consequences of your actions? (i.e. you wouldn't bet \$:1 on consequence C; you wouldn't risk a dime on $\bar{C}$.)

We consider situations where your opinions are allowed to vary; also your values. Hence, your actions. But S. would limit your actions more than objectivists.

To predict / prescribe actions, don't need numerical probabilities in general; only inequalities between probs.

Good: $P'(E|F) > P'(G|H)$ is a prob. judgment: a statement that "you" degree (or intensity) of belief in E given F (i.e. if F is assumed, or believed certain) would exceed your degree of belief in G given H.

(Doesn't require existence of numerical degrees of belief).

Good: Could a Machine Make Prob. judgments?

What side-bets might be made in chess, on the basis of different positions? A machine might lay bets on a game played by other machines.

Man is a machine that makes bets

amount of

Good: Deciding and Decisive Effort

Info & Control, 4, 271-81 1961

[Define "decision" in terms of a shift upwards in the odds a man would offer in betting that he would perform a certain act, or class of acts.]

Good: defines decision as a credibility, not an actual degree of belief. 272

Decision becomes mutually clear ~~to~~ when it becomes costly to reverse it. (Commitment; difference between "predicting I will do X" and claiming "I must now do X; I have no choice." latter may help others to make the same prediction, if that is possible desirable.

[What does it mean to say his choice is free — he "decides" — when he predicts he will choose X? Hyp: suppose that changing the payoff values, or beliefs, so as to make the consequences of another act preferable to him, will change the odds on his choice (his odds, or yours).

Threat will do this, as well as promises.

Thus, "free decisions" can still be manipulated.

See Turkey: 1960 Conclusions vs. Decisions: Technometrics

Ray, Some Notes on Pictometric Inference (

JRSS B Vol 22, 1960, No 2. 338

Whether following the minimax procedure "protects" you significantly depends on how good the minimax payoff is (and how much better than the worst, or better than the minimum associated with actions with better BGD.) What do you have to "pay" for this "protection"?

100	-100	0 / 1
-99	-99	199 / 0

i.e. what drop in BGE to get rise in LFE? or in LFE?

When you can get an acceptable minimum (perhaps by using mixed strategy) and when other, more promising actions (in terms of "best guess" or "favorable" distributions); a) are "low confidence," vague; low p
b) don't promise much more c) have "significantly" worse minima, or "reasonable minima"; then
minimax leads to acceptable result. For this to apply to strict minimax, $V^0 \text{ must} = V$, $p \approx 0$; rare.

In games, there is reason to think that LFE is "likely"; not merely prudent. But in other cases, it may seem especially unlikely.

Insurance as maximizing the probability that payoff will exceed some fixed (acceptable) figure. (To focus on this is — ? — to assume 0, 1 payoffs, depending on the "threshold").

→ see Ann. Math. Stat 28 1957 862-63

Value of info to conservative: it increases p .

(Young Doctors) (Suppose you are certain that p will go to 1).

→ see Lindley, Survey of Foundations of Statistics

Appl. Statist., Vol. 7, 1958 186-98

Is there a prior distribution that "expresses" ignorance?
i.e. that ^{as if maximizing utility} implies behavior like that of a person who feels ignorant, and is responding to his ignorance?

NO; because such a person is not acting in a way that permits us to infer any definite probs from his actions, or to describe him as maximizing expected utility. In fact, he acts so as to violate any "plausible" theory of "how to act given definite probabilities" (i.e. complete ordering of probs (qualitative probs)).

~~PROBETS~~ BETS YIELD ONLY A PARTIAL ORDERING OVER EVENTS

(BUT A RO. IS BETTER THAN NONE; and we may predict all actions, with more data than Savage requires.)

A definite (e.g. rectangular) distribution is just as bad for expressing "ignorance as a state of uncertainty" as it is for measuring "ignorance as a basis of action." vs. Jeffrey's. (Anally, Berkeley Symp. 4th, 6. 467)

(Note: complete ordering $\longleftrightarrow$ complete approximability, if we assume existence of perfect sets of cards, etc.)

Even if you had a definite Rank II prob dist over ρ , you could take 95% confidence interval (say) and mismax w.r.t. it. Now your statements might reveal definite probs but your bits would not. (Is this related to: never let $\rho \geq .95$? Not exactly: V^0 is being determined, not ρ ; $\rho = 0$?)

100. Marchak:

Either the test "How do you succeed?"
and "Will people call you reasonable?"
will depend more on the content of your beliefs
(Are they conventional? Are they "realistic"? Are
they good "when it matters"?) than on their
consistency; which is all the apions guarantee.

Other

People don't judge you by your consistency (which
is relatively hard for them to judge anyway).
Of personal interest.

Test

4-scale

I	1	0	0	0
II	0	1	0	0
III	0	0	1	0
IV	0	0	0	1
V	1	1	0	0
VI	0	0	1	1
VII	0	1	1	1
VIII	1	1	1	0

$$I = II, III = IV, V = VI$$

a) Does $I = IV$?

b) Does $VII = VIII$?

The answers: $I < IV$ and $VII > VIII$ are "inconsistent" with each other (P2) and ~~with~~ both are inconsistent with $I = II, III = IV, V = VI$ (P1 and P2).

de Jouvinal

1. Decision-making as resolving conflicts.
2. "Actions possible" depend on subject; he must be conscious of alternatives.
3. "Social order" conceived as "team problem." (stems on common interests, abstracting from conflicts).
What decisions should be made by sovereign?
4. Bernoulli: units (scale) of payoff will affect choice
(old francs vs. new francs)
5. Ignore mathematical deduction from premises; examine premises & results.
6. Which is more important to our predictions, past frequencies (e.g. on business cycle) or present expert opinion (on the impact of special circumstances on "frequency" or "likelihood"; worth of these depends on
(a) theories; b) knowledge of current circumstances.
7. Logic of opinions. (non-numerical; but completely ordered?).

8. Interpret Koopman's options in terms of gambling.
9. Suppose you, as counsellor, measure your subjects' prior probabilities and find them very mistaken, leading to disaster; you could try to change them by argument or experimental evidence; but what if you failed? Would you tell him to be consistent? Or would you to encourage him to be inconsistent? (e.g. suppose he believed Red to be "almost certain," after seeing 16 Blacks in a row at roulette; & was prepared to stake his fortune?)

Consistency is to enable prompt decision-making, not to assure ourselves of good outcome; to determine quickly what we would calculate at length to be, in our opinion (for what that is worth) is the "most promising" course; it specifies relevant considerations & prior choices to be systematically considered (speeds up deliberation, makes it more reliable).

Mathematician: "I've already solved that problem."
He explains: The psychology of mathematicians."
(Proceed systematically, slowly, but with high confidence of attaining certain result).

13. "Normative" only in the sense of "norm of consistency".

What variables is it reasonable to take into account (as "premises")? Then values of those — e.g. desires, opinions — what choices is it ^{consistent} reasonable for him to take?

What are rules of deduction from premises involving uncertainty? What sort of premises are required to determine result?

Simon: reasonable behaviour is a process of ~~thinking~~ reasoning (What men would do if they stopped to think);

(One motive: to make a decision which others on the team — esp. if they share values & opinions — will accept (or even, could predict).

Goal: a rule-book telling what you would do, given certain info.

14. Systems analysis: can use technique of chess-player;

"What would I do in his place — if I were he."

But beware.

15. Guard against ambushes (or pot-holes).

Intuitive probs "must" be "coherent", i.e.

$p_* \leq 1 - q_* = p^*$ by virtue of Koepman - Good axioms,
on "reasonable" consistent among beliefs.

But certain rules (e.g. ~~requiring~~ $\alpha > \frac{1}{2}$) lead to
incoherent "betting probs": i.e. $(p_*) + (q_*) > 1$, although
intuitive probs are reasonable.

Why rule this out?

- a) Savage axioms; but no more than $p_* + q_* < 1$.
- b) Would a "natural" modification of P2 rule this out?
- c) "Principle of coherence": for what it is worth.

If we allow this, we don't want to use betting probs to
estimate intuitive probs.

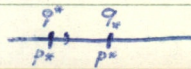
② Rule for answering question "Which do you think is more probable?"
needn't correspond to (b) rule for betting.

e.g. ② may be: take midpoint of $p_* - p^*$

② should not lead to violations of Koepman / Good axioms.

$$p^* = 1 - q_*$$

$$p_* = 1 - q^*$$



Three axioms such that inferences on events $\leq$ will satisfy Koopman axioms for relationship L . (Koopman-Coleman)

Savage axioms are too strong; give appreciable probes.

GOOD TYPE II MIN/MAX

if here probes are not all appreciable, so that $p_* < p^*$ for some events, we might call the family of distributions such that $p_*(E) \leq p(E) \leq p^*(E)$ for each event, Y^0 , the family of "reasonable" dists (all dists not ruled out by your stated body of beliefs, which obey K. axioms.)

Good: call a choice "reasonable" if for some y in Y^0 , I has a higher expectation of utility than II in terms of y (applied to both I and II); you are not required to "use" the same y in Y^0 for every choice between two alternatives.

[I have "allowed" man to use $y_1 \in Y^0$ for evaluating I and $y_2 \in Y^0$ for evaluating II : COULD I GET SAME RESULTS WITH GOOD APPROACH?]

I	1	0	0	} $I > II$ w.r.t. $(\frac{1}{3}, \lambda, \frac{2}{3}-\lambda)$ where $0 \leq \lambda < \frac{1}{3}$
II	0	1	0	
III	1	0	1	} $III > II$ w.r.t. $(\frac{1}{3}, \lambda, \frac{2}{3}-\lambda)$ where $\frac{1}{3} \leq \lambda \leq \frac{2}{3}$
IV	0	1	1	

Thus, in any paired comparison, I use a reasonable set of weights.

Maybe vagueness has an effect like static (dry) friction; certain definite push needed to "displace" one from status quo.

Will move from $(0, 0)$ to accept but on
 $\underline{\text{II}}$ only at good odds $(2, -1)$ or to bet on
 $\underline{\text{II}}$ only at good (definitely advantageous) odds
 $(-1, 2)$.

What if values of I is at least $x_{\min I}$ and
at most $x_{\max I}$?

$\bigcup_I$ 2 balls

$\bigcup_{II}$ 1R, 1B

$0 \neq R$	$1:R$	$2:R$
1	0	0
0	1	0
0	0	1
0	1	1
1	0	1
1	1	0

Type III probes: etc; at some point you should act
 "completely ignorant." (Good: increasingly vague

When will my criterion lead to Good results?

When $p y_0 + (1-p) [\alpha y_{\max} + (1-\alpha) y_{\min}]$ is in Y^0 , then
set of weights attached to each (x) will be "reasonable."

But: show that a single set of weights in Y^0 would lead
to some preferences.

see Good, RD.

Easy: for any pair of actions, the best one is best despite the
fact that some weight is given to its least favorable dist. $y_{\min}^*$;
if both are weighted $(1-p)$ by the l.f.d. corresponding to the
other action, $y: y_{\min}$, then the index of the other will be unchanged, but
the index of the former will be better or equal.

[Show for 2 actions]

and for $\alpha > 0$

convexity?

Instead of concept of p_* , $\& P^*$, could have

$\bar{p}_*(E)$, $p_*(E)$; or $p_*(E)$ for every E , but not
necessarily that $p_*(E) + p_*(\bar{E}) = 1$.

If p_* to be considered operational def. of "belief,"
then $p_* + q_*$ must = 1. If p_* to be considered op. def. of
lower prob., then $p_* + q_* \leq 1$ (otherwise relationship
wouldn't obey K. axioms).

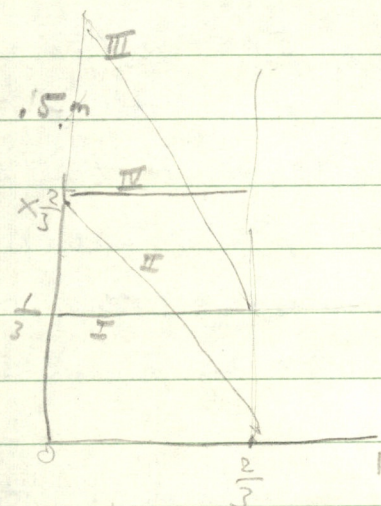
From K axioms, if $t(n) \prec E$, then, since $t(n) \succ t(n)-1$,
 $E \succ t(n)-1$, etc. So we get interval
between p_* and $(1-q_*)$.

allais: ~~the~~ one may have a special desire to make a book.

Mc: One may have a special desire to achieve an unambiguous distribution, or an explicit expectation.

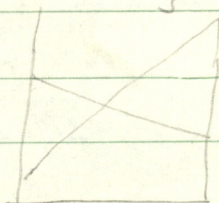
Suppose I have 9 chances out of 10 to gain 100 million francs; then I may pay more for the 10th, making a book, than for I would for the 1st (or 2d, 3d, etc. or even 9th).

1	2-10
10m	0
p5	p5
10	10
p=1	
1	0
0	1



Consider

$\frac{1}{4} - \frac{3}{4}$	$\frac{1}{5} - \frac{2}{5}$	C
A	B	
1	0	0
0	1	0



when $A \parallel B$. Here P1 might be violated

The wagueness of all measurements is relevant to decision; e. g. suppose a vital measurement of length has been conducted twice, or by two processes, or the dial read by two observers, or the unit on the scale is coarse, or the instrument is known to have a certain dist. of errors, etc.

How does relative "range of ambiguity" affect action (when none measurement is impossible)?

Which thing do we "say" is longer?

w.r.t. Which do we act "as if" it were longer?

Do we ~~act~~ act consistently with interpolation when "none length" is good and when it is bad?

All wagueness in measurement can be expressed in terms of ~~probes or in~~ "vague probes action under uncertainty", which may or may not be expressed by definite probes.

This applies also to measurement of probes.

|| Allais examples cast doubt on whether for any set of events, (n-scale) axioms are obeyed generally; possibility of alternative offering same outcome effects results. I might allow obedience for some events, but not for all. Allais has analogous but different problem.

Break down γ^0 into subsets such that within each, there is a set of states corresponding to γ 's, such that, — given that subset — you are equally willing to bet on or against each γ . Then associate with that subset, a function of its min and max expectation (e.g., min) for each action.

Now, if this function (e.g., min) is less for Π than for I for each subset of γ 's, $I > \Pi$.

If it isn't....

(Then with 4 colors: consider bet on Red vs. bet on not-Red.)

Suppose: Red, Black, Yellow, Other : 120 balls.

On: R, B, Y, Other, 30 each.

Red = Black = Yellow; = Other? Why not?

(e.g., suppose Other = Green). Why is Red < Black & Yellow; or why is Red < not-Red?

γ^0

Continue: Type II minimax

Suppose rule is: $p=0, \alpha > 0$

$I > II$

Problem: find $y \in Y^0 \ni E_y(I) > E_y(II)$

Take $y: \alpha y_{\max}^I + (1-\alpha) y_{\min}^{II}$

This will make II look $\leq$ before, and I look $\geq$ before.

Must assume this is in Y^0 : Y^0 convex?

Problem: do this for n actions. $I > II, I > III$

e.g. 3. assume $p=\alpha=0$ Type II minimax

if Y^0 is convex...

$\exists$ at least ~~one~~ $y \ni I > II (y_{\min}^{II})$ ~~$y_{\min}^I$~~

~~so there are at least~~

$$r \cdot y_{\min}^I + (1-r) y_{\min}^{II} \quad 0 \leq r \leq 1$$

likewise at least one $y \ni I > III (y_{\min}^{III})$

Take least favorable y in $Y^0 \ni y \ni$

$\max_s U(s, y)$ is minimum $y \in Y^0$

$$\begin{matrix} & I & O & O \\ \begin{matrix} I \\ O \\ O \end{matrix} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{matrix} \quad \left(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}-\lambda\right)$$

$\lambda \leq \frac{1}{3}$
 $\lambda = \frac{1}{3}$

Show that my rule: $\max X_{\min}$
is equivalent ^{To} Bayes solution against
l.f.d. in Y^0

Can randomize over all three

$$\begin{matrix} & 1 & 0 & \phi \\ \begin{matrix} 0 \\ 2 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \end{pmatrix} \end{matrix}$$

Consider y 's for which it makes a difference between I and II ; lump together all y 's that make a given difference between I and II .

Consider each

Problem: Apply Arrow Theorem to V^0 , show that Hurning rule applies.

a) e.g. if I would be indifferent between betting on or against each composition of the urn backbone

$(\frac{1}{2}, 1, 1-\lambda)$, then I need pay attention only to min and max expectations for each action.

b) Suppose there is a set of y 's ≥ 2 such that I am indifferent between betting on or against each one; then consider only min and max for this set; given this set.

$$p y_0 + (1-p) [\alpha y_{\max} + (1-\alpha) y_{\min}] (x)$$
$$p y_0 + (1-p) [f(y_{\max}, y_{\min})]$$

STRATEGY

The difference between the systems analysis problem and the operational decision problem is that latter is confined to choice among given alternatives and given info; and ⁱⁿ the former, the main task is not analysis but design & invention of alternatives, + purchase of info.

Current choices ^{R+D, planning, procurement} affect later alternatives ^{+ info} available to operators ^{and flexible objectives}. But given uncertainty about later objectives, environment, info, major criterion is to broaden field of choice, provide alternatives + info.

Why is this not done? Incentives to restrict alternatives; cost of innovation, flexibility; failure to "see" uncertainty.

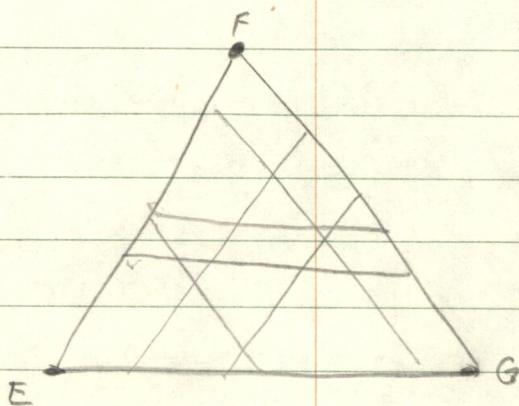
In practical applications: ~~Make choices that~~
~~imply~~ Make choices that imply certain
inequalities (or equalities) ^{on utilities & preferences} to be consistent w.r.t.
Bernoulli theorem. Then show that certain
choices (hopefully; all but one; not the programmed
system) ~~must~~ must be rejected, to allow
consistency with Bernoulli theorem (i.e. to accept
them would be to allow a pattern of choices
for which no numbers would exist satisfying the
Bernoulli theorem.)

Suppose we express judgments of confidence
in prob judgments; and have a set Y^0 of
reasonable diets. Also, y_0 .

Rule (1): If $I > II$ for every y in Y^0 ,
then $I > II$. Likewise, for $I = II$.

Rule (2): For each $y^i \in Y^0$, compute x^i
for I and II .

Each y^i can be identified with a state such
that if it obtained, one's judgment would be y^i :
(e.g. a given proportion of balls in urn; having
excluded some proportions as unreasonable) (Koopman).



$A \cup E \supseteq B$

$A \parallel B$

$B \cup E \supseteq A$

$A \parallel C$

$C \cup E \parallel A$

$B \parallel C$

$A \cup E \parallel C$

$E = 10$ heads is a new

A, B, C wide, equally wide

If this were put in terms of intervals, we couldn't get back this information.

"same" from "vague" judgments; of no more use than knowing (linear) utility of small sums.

Suppose we find 50 axioms obeyed for certain events and for certain money outcomes.

Hyp: they will be obeyed for all events involving these ("small") outcomes.

But for other, "larger" outcomes, both utility and vagueness considerations may "distort" choices: the first, to lead to violations of ^{Bernoulli's hyp.} axioms for all events, with ^{"t.i."} these money "consequences," (and any given probes), the second for some (vague) events, even when utilities (non-linear) are "measured."

If one obeys VN-M axioms for some P-function (e.g. Koopman) then he will obey 5 axioms, and will "reveal" this same P-function.

If one obeys 5 axioms, then you will obey VN-M axioms only w.r.t. Savage probes, not w.r.t. any other that disagree.

If you don't obey Savage Bernoulli theorem, you must be violating Savage axioms. SHOW DIRECT CONTRADICTIONS OF BERNOULLI THEOREM.

de Finetti: Bayesian Horizons
Satané:

Savage: formulate some rules of conduct that I would like never to violate: A BASIS for REJECTING some actions: if they "logically conflict" with our definite opinions and values; application: find definite judgments such that only one of a finite set of actions is compatible.

(e.g. under certainty: compare the finite set of consequences: P3...),

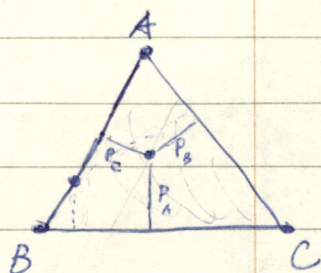
(No use if you can't tell which judgments are "basic," "definite"; what judgments should be used for inference? Which should be changed when there are inconsistencies?)

e.g. formulate alternatives to PROGRAM, show that program must be rejected (Goal of systems analysis); on basis of CLEAR, DEFINITE judgments and acceptable criteria.

What if "small sums" are defined by approximately
linear utility: i.e., ^{VN-M} axioms obeyed for money.

Within this same region, Savage-de Finetti axioms should be obeyed; for measurement of "best guess" probs. (But then, couldn't tell "best guess" from

Grillboard



$$P_A + P_B + P_C = 1$$

All prospects are points in triangle; pts. on a segment involve just two events.

Can have indifference curves over triangle.

But if A+B are pts. on an indifference curve, "it is natural to postulate that the ~~total~~ equivalence of A and B entails that of every [risk] combination of A and B [Postulate...]. As a result, the indifference lines are necessarily straight; moreover, they are parallel. One is then led to the classical calculus of Moral Expectation." [Kendall & Savage result]. [on Simple Dictatorship].

[I accept for "definite" risks; but mixtures of ambiguous risks may be preferred to either].

